DOMINATION NUMBER TO THE TRANSFORMATIONS OF STAR GRAPH

Stephen John.B (1) **Krishna Girija.B** (2)

Associate professor Scholar (Reg.No:20193013545206)

P.G and Research Department of Mathematics,

Annai Velankanni College(Accredited with B++ by NAAC)

Tholayavattam, Kanyakumari District – 629 157

Affiliated to Manonmaniam Sundaranar University, Abishekapatti Tirunelveli-629012,TamilNadu, India

Introduction:

Graph Theory is one of the best application oriented area in mathematics. In domination many research articles have been published. In 1952, Cladue Berge defined the concept of domination number and Ore introduced the term in 1962. In 1977, Cockayne and Hedetniemi introduce the notation $\gamma(G)$ *to denote the domination number. The transformation graph* G^{xyz} *of G was introduced by Wu and Meng in 2001. Graph transformation is a vast developing research area in Graph Theory. For each graph we can obtain eight transformation graphs by using the symbols* $(+,-)$ *for the values* x, y, z. *This application is used in Theory of Network Communication and Radiology.*

Abstract:

In this paper, we have studied the dominating set and domination number of star graph and its various transformation.

Key words:

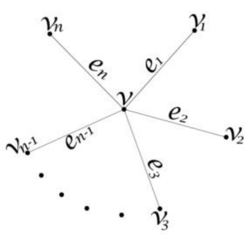
Domination, Domination Number, Graph Transformation.

Definition: 1.1

A **graph** G is an ordered pair (V(G),E(G)) consisting of a non empty set V(G) of vertices and a set E(G) , disjoint from V(G) of edges together with an incidence function ψ_G that associates with each edge of G is an unordered pair of vertices of G.

Definition: 1.2

$K_{1,n}$ is called a **star graph** for $n \geq 1$.



(Figure: 1)

Definition: 1.3

Let G = (V(G),E(G)) be a graph x, y, z be three variables taking values + or $-$. The vertex set of the **transformation graph** $\boldsymbol{G^{xyz}}$ of G is V(G) ∪ E(G) and each pair of

(α, β) ∈ V(G^{xyz}) is adjacent if and only if the following holds.

(i). α, β ∈ V(G), α *and* β are adjacent in G if x = + ; α *and* β are not adjacent in G if $x = -$.

(ii). α, β ∈ E(G), α *and* β are adjacent in G if y = + ; α *and* β are not adjacent in G if $y = -$.

(iii). $\alpha \in V(G), \beta$ ∈ E(G), α *and* β are adjacent in G if z = + ; α *and* β are not adjacent in G if $z = -$.

Definition: 1.4

A subset S of vertices in a graph G is said to be a **dominating set** if every vertex v ∈ V-S is adjacent to at least one element of S.

The number of vertices in a minimum dominating set of G is called a **domination number** of G. It is denoted by $\gamma(G)$.

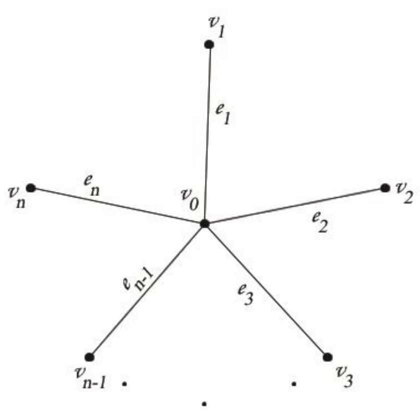
Theorem: 2.1

Let G be any star graph with n + 1 vertices, then $\gamma(G^{-+-})$ = 3.

Proof:

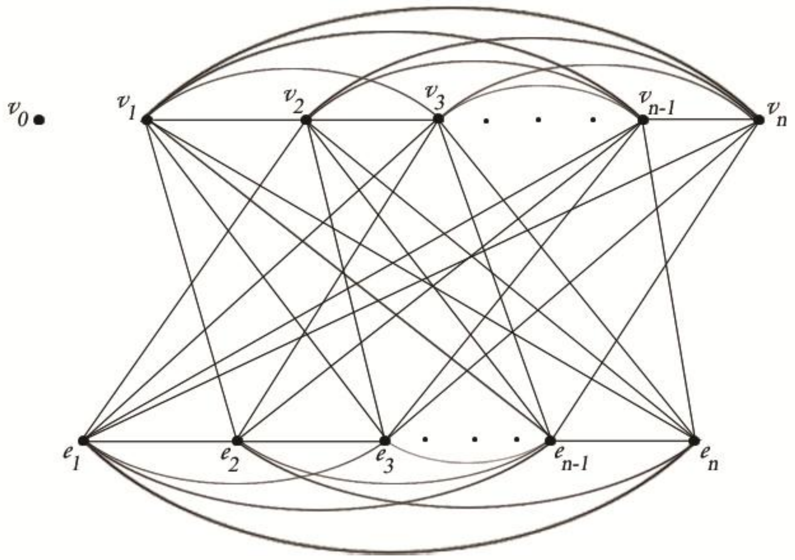
Let G = $K_{1,n}$ be a star graph with n + 1 vertices, V(G) = $\{v_0, v_1, v_2, \ldots, v_n\}$ and

E(G) = $\{e_1, e_2, e_3, \ldots, e_n\}$ is the vertex and edge set of G respectively.



(Figure: 2)

Let G^{-+-} be a transformation of G and V(G^{-+-}) = $\{v_0, v_i, e_i /$ i = 1,2,3,...,n$\}$.



(Figure: 3)

In G^{-+-} , N(v_i) = V(G^{-+-}) – $\{v_0, e_i\}$

N(e_i) = V(G^{-+-}) – $\{v_0\}$

Hence, $\{v_i /$ i = 1,2,3,...,n$\}$, $\{e_i /$ i = 1,2,3,...,n$\}$ form a clique of order n in G^{-+-}.

That is, N(v_i, e_i) = V(G^{-+-}) – $\{v_0\}$ for all i. [since d(v_0) = 0]

Let D_i = $\{(v_i, e_i) \cup \{v_0\}\} /$ i = 1,2,3,...,n, V(G^{-+-}) – D_i is adjacent with D_i for all i. Hence, each D_i is a dominating sets of G^{-+-} with minimum cardinality and $|D_i|$ = 3 for all i.

Therefore, $\gamma(G^{-+-})$ = 3.

Theorem: 2.2

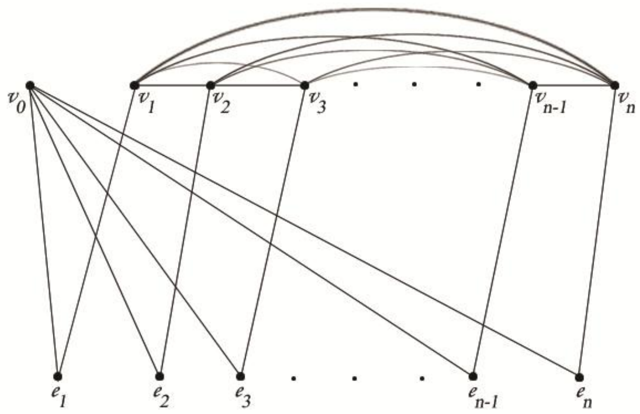
Let G be any star graph with n + 1 vertices , then $\gamma(G^{--+})$ = 2.

Proof:

Let G = $K_{1,n}$ be a star graph with n + 1 vertices and V(G) = $\{v_i /$ i = 1,2,3,...,n$\}$,

E(G) = $\{e_i /$ i = 1,2,3,...,n$\}$ where $e_i = v_0 v_i$ and d(v_0) = n , d(v_i) = 1 for all i.

Let G^{--+} be a transformation of G.



(Figure: 4)

In G^{--+} , N(v_i) = {v_j, e_i/ j = 1,2,3,…,n} and

N(e_i) = (v_i) / i = 1,2,3,…,n

Clearly, {v_i/ i = 1,2,3,…,n} form a clique in G^{--+} and d(e_i) = 2 for all i.

Let D_i = {v_0, v_i/ i = 1,2,3,…,n} are the dominating sets of G^{--+} with minimum cardinality.

Therefore, $|D_i|$ = 2 for all i.

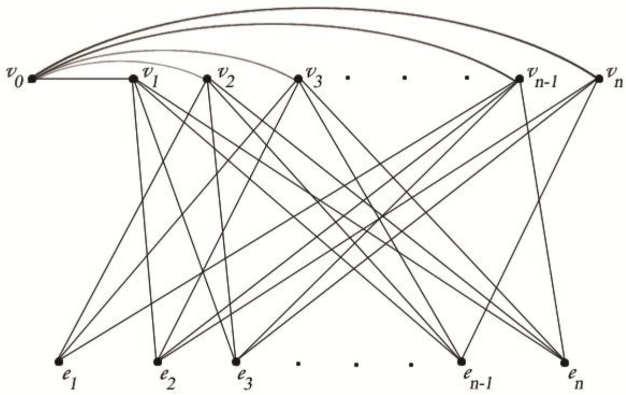
Hence, $\gamma(G^{--+})$ = 2.

Theorem: 2.3

Let G be any star graph with n + 1 vertices, then $\gamma(G^{+--})$ = 2.

Proof:

Let G be any star graph with n + 1 vertices and G^{+--} is one of its transformation.



(Figure: 5)

In G^{+--} , V(G^{+--}) = $\{v_0, v_i, e_i$ / i = 1,2,3,...,n}

N(v_0) = $\{v_i$/ i = 1,2,3,...,n}

N(v_i) = $\{v_0, e_j$ / j = 1,2,3,...,n and i ≠ j} for all i.

N(e_i) = $\{v_j$ / j = 1,2,3,...,n and i ≠ j} for all i.

Clearly, D_i = { v_i, e_i / i = 1,2,3,...,n} are the dominating sets of G^{+--} with minimum cardinality.

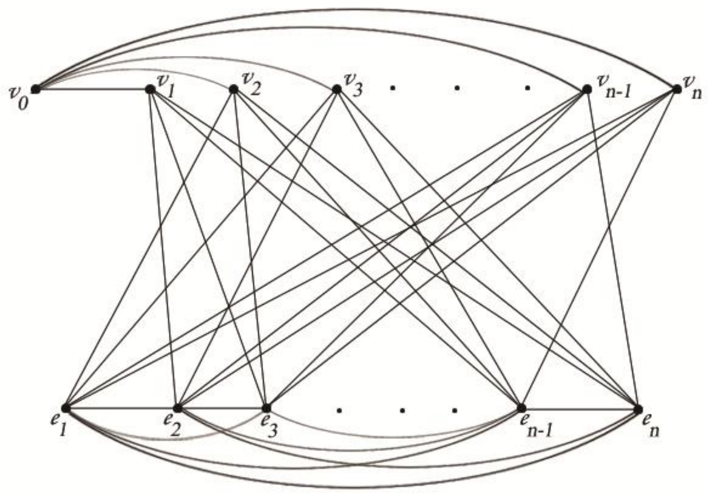
Therefore, $\gamma(G^{+--})$ = 2.

Theorem: 2.4

Let G be any star graph with n + 1 vertices, then $\gamma(G^{++-})$ = 2.

Proof:

Let G be any star graph with n + 1 vertices and G^{++-} is one of its transformation.



(Figure: 6)

In figure, N(v_i) = $\{v_o, e_j$ / j = 1,2,3,...,n and i ≠ j} for all i.

N(e_i) = $\{v_j , e_j$ / j = 1,2,3,...,n and i ≠ j} for all i.

Clearly, D_1 = $\{v_i, e_i$ / i = 1,2,3,...,n} and D_2 = $\{v_0, e_i$ / i = 1,2,3,...,n} are the required dominating sets of G^{++-} with minimum cardinality.

Therefore, each dominating sets has two elements.

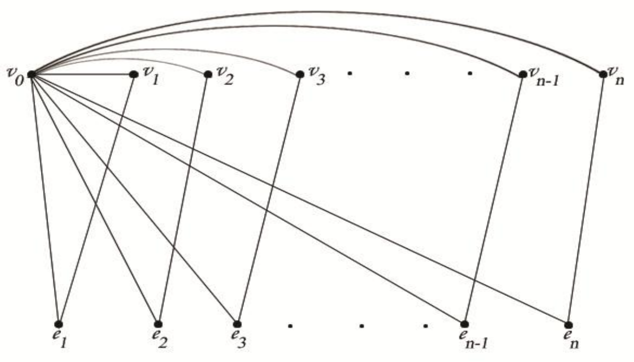
Hence, $\gamma(G^{++-})$ = 2.

Theorem: 2.5

Let G be any star graph with n + 1 vertices, then $\gamma(G^{+-+})$ = 1.

Proof:

Let G be any star graph with n + 1 vertices and let G^{+-+} be one of its transformation in G.



(Figure: 7)

In G^{+-+} , V(G^{+-+}) = $\{v_0, v_i, e_i$ / i = 1,2,3,...,n}

N(v_0) = $\{v_i, e_i$ / i = 1,2,3,...,n}

Clearly, $\{v_0\}$ is dominates all the elements in G^{+-+}.

Therefore, $\{v_0\}$ is only minimum dominating set of G^{+-+}.

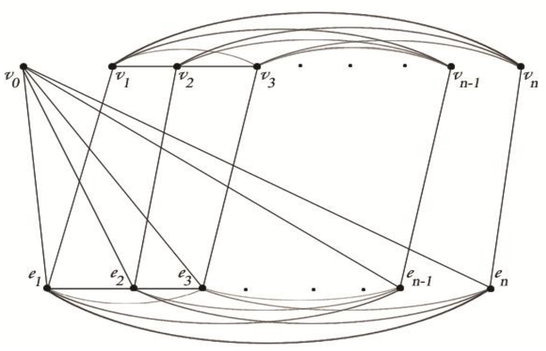
Hence, $\gamma(G^{+-+})$ = 1.

Theorem: 2.6

Let G be any star graph with n + 1 vertices, then $\gamma(G^{-++})$ = 2.

Proof:

Let G be any star graph with n + 1 vertices and G^{-++} is one of its transformation.



(Figure: 8)

In figure, $N(v_0) = \{e_i \ / \ i = 1,2,3,...,n\}$

$N(v_i) = \{v_j, e_i \ / \ j = 1,2,3,...,n\}$

$N(e_i) = \{v_0, v_i, e_j \ / \ j = 1,2,3,...,n\}$

Then, $\{v_0, e_i \ / \ i = 1,2,3,...,n\}$, $\{v_i \ / \ i = 1,2,3,...,n\}$ form a clique in G^{-++}.

Let $D_1 = \{v_i, e_i \ / \ i = 1,2,3,...,n\}$ and $D_2 = \{v_0, v_i \ / \ i = 1,2,3,...,n\}$ are the required dominating sets of G^{++-} with minimum cardinality.

Hence, $\gamma(G^{-++}) = 2$.

References:

[1]. *Angel Jebitha. M . K and Paulraj Josheph. J, The domination subdivision Number of Transformation Graphs, International Journal of Mathematical Sciences and Applications 3(1) (2013).*

[2]. *Basavanagoud. B,Keerthi Mirajkarandshripurnamalghan, Transversability and Planarity of the transformation graph G^{xyz} Proceedings of International Conference on Graph Theory and Applications, Amirtha School, 2009.*

[3]. *Bondy.J.A and Murty. U.S.R, Graph Theory with Applications,NorthHolland, New York*

[4]. *Harary.F, Graph Theory, Narosa publishing house (1969).*

[5].*Hedetniemi.S.T and Laskar.R.C, "Bibliography on domination in graphs and somebasic definitions of domination parameters" Discrete Mathematics.(1990).*

[6]. *Kulli.V.R ,Theory of domination in graphs, Vishwa international publications, Gulburga, India (2010).*

[7].*Wu.B, Meng. J, "Basic properties of total transformation graphs" J.Math study 34 (2) (2001).*

[8].*Wu.B, Zhang. L, Zhang. Z, The Transformation Graph G^{xyz} when $xyz = -++$ Discrete Math.,296(2005).*