

NANO REGULAR GENERALIZED STAR STAR b - CLOSED SETS IN NANO BITOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce the concept of Nano Regular Generalized star star b - closed sets in Nano Bitopological Spaces and we discuss some basic properties also reformulate the axioms, in particular we investigate the characteristics of $\tau_1\tau_2 Nrg^{**}b$ - closed sets whose results are given in detail.

Key words $Nrg^{**}b$ – closed sets , $\tau_1\tau_2 Nrg^{**}b$ -closed sets.

1.INTRODUCTION

General topology is vast has many different ramifications and interactions. In the analysis and visualization of all types of field data, topology based methods play a predominant role. The notions of closed sets are the simple concepts and beginning point of various great theories, which is exactly investigated in topology often. The most basic and conventional divisions of topology is point set topology, which establishes the basic aspects of topology and also highlights the regular and normal spaces. Pawlak introduced the rough topology[4]

Lellish Thivagar[1] introduced a nano topological space respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region . The elements of a Nano topological space with are called the Nano open set. It originates from the Greek word ' Nanos' which means 'dwarf' in its modern scientific sense, an order to magnitude-one billionth. The Topology is named as Nano topology so because of its size, since it has at most five elements.Vasantha kannan and Indirani introduced the Concept of Nano Regular Generalized star star b- closed sets in Nano topological Spaces. In this paper, we discuss the $\tau_1\tau_2$ Nano regular generalized star star b-Closed sets in Nano bitopological spaces.

2.PRELIMINARIES

Definition-2.1[2]Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes.Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by $x \in U$.

(ii)The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}$ where $R(x)$ denotes the equivalence class determined by $x \in U$.

(iii) The boundary region of X with respect to R is set of all objects, which can be classified neither in X nor not in X with respect to R and it is denoted by $B_R(X)$.That is, $B_R(X) = U_R(X) - L_R(X)$.

Property-2.2[2] If (U, R) is an approximation space and $X, Y \subseteq U$, then

$$(i) L_R(X) \subseteq X \subseteq U_R(X)$$

$$(ii) L_R(\emptyset) = U_R(\emptyset) = \emptyset$$

$$(iii) L_R(U) = U_R(U) = U$$

$$(iv) U_R(X \cup Y) = U_R(X) \cup U_R(Y)$$

$$(v) U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$$

$$(vi) L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$$

$$(vii) L_R(X \cap Y) = L_R(X) \cap L_R(Y)$$

$$(viii) L_R(X) \subseteq L_R(Y) \text{ and } U_R(X) \subseteq U_R(Y) \text{ whenever } X \subseteq Y$$

$$(ix) U_R(X^c) = [L_R(X)]^c \text{ and } L_R(X^c) = [U_R(X)]^c (x) U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$$

$$(xi) L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$$

Definition-2.3[2] Let U be a non-empty, finite universe of objects and R be an equivalence relation on U . Let $X \subseteq U$. $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$. Then $\tau_R(X)$ satisfies the following axioms:

(i) U and $\emptyset \in \tau_R(X)$.

(ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$. That is, $\tau_R(X)$ is a topology on U , called as the Nano topology with respect to X . Elements of Nano topology are known as the Nano open sets in U and $(U, \tau_R(X))$ is called the Nano topological space. Elements of $(\tau_R(X))^c$ are called as Nano closed sets.

Definition-2.4[2] Let $(U, \tau_R(X))$ be a Nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

(i) The Nano interior of A is defined as the union of all Nano open subsets of A and it is denoted by $Nint(A)$. That is, $Nint(A)$ is the largest Nano open subset of A .

(ii) The Nanoclosure of A is defined as the intersection of all Nano closed sets containing A and is denoted by $Ncl(A)$. That is $Ncl(A)$ is the smallest Nano closed set containing A .

Definition-2.5 $\tau_1 \tau_2$ Nano regular open in X if $A = \tau_1 Nint[\tau_2 Ncl(A)]$

Definition-2.6 $\tau_1 \tau_2$ Nano pre open in X if $A \subseteq \tau_1 Nint[\tau_2 Ncl(A)]$

Definition-2.7 $\tau_1 \tau_2$ Nano semi open in X if $A \subseteq \tau_1 Ncl[\tau_2 Nint(A)]$

Definition-2.8 $\tau_1 \tau_2$ Nano b open in X if $A \subseteq \tau_2 Ncl(\tau_1 Nint(A)) \cup \tau_1 Nint(\tau_2 Ncl(A))$.

Definition-2.9 $\tau_1 \tau_2$ Nano α open in X if $A \subseteq \tau_1 Nint(\tau_2 Ncl(\tau_1 Nint(A)))$.

Definition-2.10 $\tau_1 \tau_2$ Ng closed in X , if $\tau_2 Ncl(A) \subseteq U$ whenever $A \subseteq U$ and U is Nano open in τ_1 .

Definition-2.11 $\tau_1 \tau_2$ Ng* closed in X , if $\tau_2 Ncl(A) \subseteq U$ whenever $A \subseteq U$ and U is Nano g-open in τ_1 .

Definition-2.12 $\tau_1 \tau_2$ Ng α closed in X , if $\tau_2 N\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is Nano α open in τ_1 .

Definition-2.13 $\tau_1 \tau_2$ Ngp closed in X , if $\tau_2 Npcl(A) \subseteq U$ whenever $A \subseteq U$ and U is Nano open in τ_1 .

Definition-2.14 $\tau_1 \tau_2$ Ngpr closed in X , if $\tau_2 Npcl(A) \subseteq U$ whenever $A \subseteq U$ and U is Nano regular open in τ_1 .

Definition-2.15 $\tau_1\tau_2Nrg^*$ closed in X , if $\tau_2Ncl(A) \subseteq U$ whenever $A \subseteq U$ and U is Nano regular open in τ_1 .

Definition-2.16 $\tau_1\tau_2Nrg^*$ b -closed in X , if $\tau_2Nbcl(A) \subseteq U$ whenever $A \subseteq U$ and U is Nrg^* open in τ_1 .

Definition-2.17 Let A be a subset of a Nano bitopological space $(U, \tau_1\tau_2(X))$, the intersection of all $\tau_1\tau_2Nrg^*$ b -closed sets containing A is called τ_2Nrg^* b -closure of A . That is $\tau_2Nrg^*bcl(A) = \cap \{F : F \text{ is } \tau_2Nrg^*b\text{-closed in } U, A \subseteq F\}$.

3. $\tau_1\tau_2Nrg^*$ b –CLOSED SETS IN NANO BITOPOLOGICAL SPACES

Definition-3.1 A Subset A of a Nano bitopological space $(U, \tau_1\tau_2(X))$ is called Nano regular generalized star b -closed set (briefly $\tau_1\tau_2Nrg^*$ b -closed) if $\tau_2Nrg^*bcl(A) \subseteq Y$ whenever $A \subseteq Y$ and Y is Nano open in $(U, \tau_1(X))$. The collection of all $\tau_1\tau_2Nrg^*$ b -closed subsets of U is denoted by $\tau_1\tau_2NRG^*BC(U, X)$.

Theorem 3.2 Every τ_2 Nano closed set is $\tau_1\tau_2Nrg^*$ b -closed.

Proof. Let A be τ_2 Nano closed in $(U, \tau_1\tau_2(X))$ and Y be τ_1 Nano open in $(U, \tau_1\tau_2(X))$ such that $A \subseteq Y$. Since A is τ_2 Nanoclosed, $\tau_2Ncl(A) = A \subseteq Y$. Since “every τ_2 Nano closed set is $\tau_1\tau_2Nrg^*$ b -closed”, $\tau_2Nrg^*bcl(A) \subseteq \tau_2Ncl(A) = A$. Therefore $\tau_2Nrg^*bcl(A) \subseteq Y$. Hence A is $\tau_1\tau_2Nrg^*$ b -closed.

Theorem 3.3

- (i) Every $\tau_1\tau_2$ Nano semi closed set is $\tau_1\tau_2Nrg^*$ b -closed.
- (ii) Every $\tau_1\tau_2$ Nano pre closed set is $\tau_1\tau_2Nrg^*$ b -closed.
- (iii) Every $\tau_1\tau_2$ Nano α closed set is $\tau_1\tau_2Nrg^*$ b -closed.
- (iv) Every $\tau_1\tau_2$ Nano regular closed set is $\tau_1\tau_2Nrg^*$ b -closed.

Proof. (i) Let A be $\tau_1\tau_2$ Nano semi closed in $(U, \tau_1\tau_2(X))$ and Y be Nano τ_1 open in $(U, \tau_1\tau_2(X))$ such that $A \subseteq Y$. Since A is τ_2 Nanosemi closed, $\tau_2Nscl(A) = A \subseteq Y$. Since “every $\tau_1\tau_2$ Nano semi closed set is $\tau_1\tau_2Nrg^*$ b -closed”, $\tau_2Nrg^*bcl(A) \subseteq \tau_2Nscl(A) = A$. Therefore $\tau_2Nrg^*bcl(A) \subseteq Y$. Hence A is $\tau_1\tau_2Nrg^*$ b -closed.

Proof for the results (ii) to (iv) are similar to (i).

Remark 3.4 Reverse implications of the above theorems need not be true as seen from the following example.

Example 3.5: $U = \{a, b, c, d\}$ with $U/R_1 = \{\{a\}, \{b, c\}, \{d\}\}$ and $X = \{a, c\} \subseteq U$. Then the Nano topology $\tau_1(X) = \{U, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $U/R_2 = \{\{b\}, \{a, c\}, \{d\}\}$ and $X = \{a, b, c\} \subseteq U$. Then the Nano topology $\tau_2(X) = \{U, \emptyset, \{a, b, c\}\}$.

The sets $\{a, d\}$ and $\{b, d\}$ are $\tau_1\tau_2Nrg^{**}b$ -closed but not τ_2 Nano closed.

The sets $\{a, b, d\}$ and $\{c, d\}$ are $\tau_1\tau_2Nrg^{**}b$ -closed but not τ_2 Nano semi closed.

The sets $\{a, d\}$ and $\{b, c, d\}$ is $\tau_1\tau_2Nrg^{**}b$ -closed but not τ_2 Nano pre closed.

The sets $\{b, d\}$ and $\{a, c, d\}$ are $\tau_1\tau_2Nrg^{**}b$ -closed but not τ_2 Nano α closed.

The sets $\{d\}$ and $\{c, d\}$ are $\tau_1\tau_2Nrg^{**}b$ -closed but not τ_2 Nano regular closed.

Theorem 3.6 Every $\tau_1\tau_2$ Nano g -closed set is $\tau_1\tau_2Nrg^{**}b$ -closed.

Proof. Let A be $\tau_1\tau_2$ Nano g -closed in $(U, \tau_1\tau_2(X))$ and Y be τ_1 Nano open in $(U, \tau_1\tau_2(X))$ such that $A \subseteq Y$. Since A is τ_2 Nano g -closed, $\tau_2Ncl(A) \subseteq Y$. Since “every $\tau_1\tau_2$ Nano closed set is $\tau_1\tau_2Nrg^*b$ -closed”, $\tau_2Nrg^*bcl(A) \subseteq \tau_2Ncl(A) \subseteq Y$. Therefore $\tau_1\tau_2Nrg^*bcl(A) \subseteq Y$. Hence A is $\tau_1\tau_2Nrg^{**}b$ -closed.

Theorem 3.7 Every $\tau_1\tau_2$ Nano g^* -closed set is $\tau_1\tau_2Nrg^{**}b$ -closed.

Proof. Let A be $\tau_1\tau_2$ Nano g^* -closed in $(U, \tau_1\tau_2(X))$. Therefore $\tau_2Ncl(A) \subseteq Y$ whenever $A \subseteq Y$ and Y is τ_1 Nano g -open in $(U, \tau_1\tau_2(X))$. As every τ_1 Nano open set is τ_1 Nano g -open, we get $\tau_2Ncl(A) \subseteq Y$ whenever $A \subseteq Y$ and Y is τ_1 Nano open in $(U, \tau_1\tau_2(X))$. But it is true that “every $\tau_1\tau_2$ Nano closed set is $\tau_1\tau_2Nrg^*b$ -closed”, $\tau_2Nrg^*bcl(A) \subseteq \tau_2Ncl(A) \subseteq Y$. Therefore $\tau_1\tau_2Nrg^*bcl(A) \subseteq Y$. Hence A is $\tau_1\tau_2Nrg^{**}b$ -closed.

Theorem 3.8 Every $\tau_1\tau_2$ Nano $g\alpha$ -closed set is $\tau_1\tau_2Nrg^{**}b$ -closed.

Proof. Let A be $\tau_1\tau_2$ Nano $g\alpha$ -closed in $(U, \tau_1\tau_2(X))$. Therefore $\tau_2Nacl(A) \subseteq Y$ whenever $A \subseteq Y$ and Y is τ_1 Nano α -open in $(U, \tau_1\tau_2(X))$. As every τ_1 Nano open set is τ_1 Nano α -open, we get $\tau_2Nacl(A) \subseteq Y$ whenever $A \subseteq Y$ and Y is τ_1 Nano open in $(U, \tau_1\tau_2(X))$. But it is true that “every $\tau_1\tau_2$ Nano α -closed set is $\tau_1\tau_2Nrg^*b$ -closed”, $\tau_2Nrg^*bcl(A) \subseteq \tau_2Nacl(A) \subseteq Y$. Therefore $\tau_1\tau_2Nrg^*bcl(A) \subseteq Y$. Hence A is $\tau_1\tau_2Nrg^{**}b$ -closed.

Remark 3.9 Reverse implications of the above theorems need not be true as seen from the following example

Example 3.10 Let $U = \{a, b, c, d\}$ with $U/R_1 = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, b\} \subseteq U$. Then the Nano topology $\tau_1(X) = \{U, \emptyset, \{a, b\}\}$ and $U/R_2 = \{\{b\}, \{a, c\}, \{d\}\}$ and $X = \{a, b, c\} \subseteq U$. Then the Nano topology $\tau_2(X) = \{U, \emptyset, \{a, b, c\}\}$.

The set $\{a\}$ is $\tau_1\tau_2Nrg^{**}b$ -closed but not $\tau_1\tau_2$ Nano g -closed.

The sets $\{b, d\}$ and $\{b\}$ are $\tau_1\tau_2Nrg^{**}b$ -closed but not $\tau_1\tau_2$ Nano g^* -closed.

The set $\{a, d\}$ is $\tau_1\tau_2Nrg^{**}b$ -closed but not $\tau_1\tau_2$ Nano $g\alpha$ -closed.

Theorem 3.11 Every $\tau_1\tau_2$ Nano gp -closed set is $\tau_1\tau_2Nrg^{**}b$ -closed.

Proof. Let A be $\tau_1\tau_2$ Nano gp -closed in $(U, \tau_1\tau_2(X))$ and Y be τ_1 Nano open in $(U, \tau_1\tau_2(X))$ such that $A \subseteq Y$. Since A is $\tau_1\tau_2$ Nano gp -closed, $\tau_2Npcl(A) \subseteq Y$. Since “every $\tau_1\tau_2$ Nano pre

closed set is $\tau_1\tau_2Nrg^*b$ -closed", $\tau_2Nrg^*bcl(A) \subseteq \tau_2Npcl(A) \subseteq Y$. Therefore $\tau_1\tau_2Nrg^*bcl(A) \subseteq Y$. Hence A is $\tau_1\tau_2Nrg^{**}b$ -closed.

Remark 3.12 Reverse implication of the above theorem need not be true as seen from the following example.

Example 3.13 $U = \{a, b, c, d\}$ with $U/R_1 = \{\{a\}, \{b, c\}, \{d\}\}$ and $X = \{a, c\} \subseteq U$. Then the Nano topology $\tau_1(X) = \{U, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $U/R_2 = \{\{b\}, \{a, c\}, \{d\}\}$ and $X = \{a, b, c\} \subseteq U$. Then the Nano topology $\tau_2(X) = \{U, \emptyset, \{a, b, c\}\}$. Then the sets $\{a, b, d\}$ and $\{b, d\}$ are $\tau_1\tau_2Nrg^{**}b$ -closed but not $\tau_1\tau_2Nanogp$ -closed.

Remark 3.14 The $\tau_1\tau_2Nrg^{**}b$ -closed set is independent with the Nano gpr -closed set and Nano rg^* -closed set as shown by the following example.

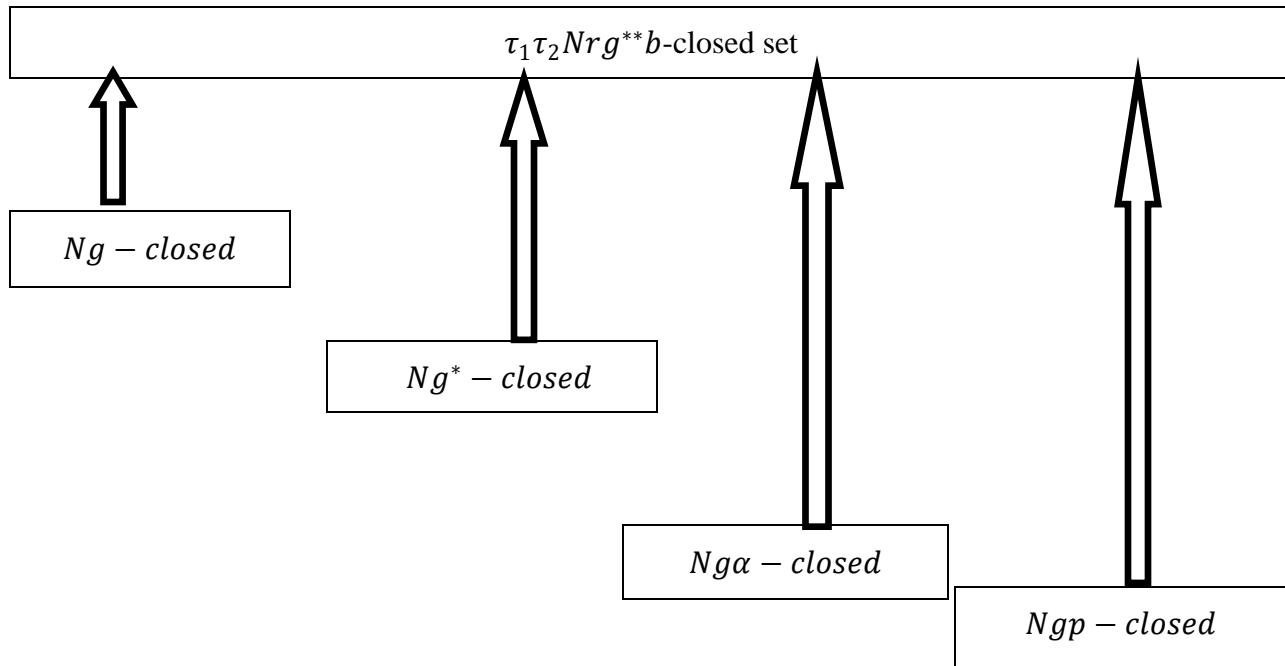
Example 3.15 $U = \{a, b, c, d\}$ with $U/R_1 = \{\{a\}, \{b, c\}, \{d\}\}$ and $X = \{a, c\} \subseteq U$. Then the Nano topology $\tau_1(X) = \{U, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $U/R_2 = \{\{b\}, \{a, c\}, \{d\}\}$ and $X = \{a, b, c\} \subseteq U$. Then the Nano topology $\tau_2(X) = \{U, \emptyset, \{a, b, c\}\}$.

The set $\{a, d\}$ is $\tau_1\tau_2Nrg^{**}b$ -closed but not $\tau_1\tau_2Nano gpr$ -closed and the set $\{a, b\}$ and $\{b, c\}$ is $\tau_1\tau_2Nano gpr$ -closed but not $\tau_1\tau_2Nrg^{**}b$ -closed.

Also the set $\{c, d\}$ is $\tau_1\tau_2Nrg^{**}b$ -closed but not $\tau_1\tau_2Nano rg^*$ -closed and the set $\{a, b, c\}$ is $\tau_1\tau_2Nano rg^*$ -closed but not $\tau_1\tau_2Nrg^{**}b$ -closed.

Diagram 3.16

*Implications of $\tau_1\tau_2Nrg^{**}b$ -closed set*



4.CHARACTERISTICS OF $\tau_1\tau_2Nrg^{**}b$ -CLOSED SETS

Theorem 4.1 For a $\tau_1\tau_2Nrg^{**}b$ -closed set A , $\tau_2Nrg^*bcl(A) \cap A^c$ contains no non-empty Nano closed set, and the converse is true if the intersection of Nano closed set and a $\tau_1\tau_2Nrg^{**}b$ -closed set is a Nano closed set.

Proof. Given that A is $\tau_1\tau_2Nrg^{**}b$ -closed. Let U' be a Nano closed set in U such that $\tau_2Nrg^*bcl(A) \cap A^c \supseteq U' \dots \dots \dots (1)$

To Prove. $U' = \emptyset$. Since A is $\tau_1\tau_2Nrg^{**}b$ -closed, $\tau_2Nrg^*bcl(A) \subseteq Y$ whenever $A \subseteq Y$ and Y is Nano open in U . From (1) $U' \subseteq A^c \Rightarrow U' \subseteq U - A \Rightarrow A \subseteq U - U'$. Therefore $\tau_2Nrg^*bcl(Z) \subseteq U - U'$, since $U - U'$ is Nano open and A is $Nrg^{**}b$ -closed. Therefore $U' \subseteq U - \tau_2Nrg^*bcl(A)$. Again from (1) $U' \subseteq \tau_2Nrg^*bcl(A)$. Therefore $U' \subseteq \tau_2Nrg^*bcl(A) \cap (U - \tau_2Nrg^*bcl(A))$. But it is obvious that $\tau_2Nrg^*bcl(A)$ and $U - \tau_2Nrg^*bcl(A)$ are disjoint. Therefore $U' = \emptyset$.

Conversely, assume that $\tau_2Nrg^*bcl(A) \cap A^c$ contains no non-empty Nano closed set.

To prove. A is $\tau_1\tau_2Nrg^{**}b$ -closed in U .

Let $A \subseteq Y$ and Y is Nano open in U . Suppose that $\tau_2Nrg^*bcl(A) \not\subseteq Y$,

$\tau_2Nrg^*bcl(A) \cap Y^c$ is a non-empty Nano closed set of $\tau_2Nrg^*bcl(A) \cap A^c$. Which is a Contradiction. Therefore $\tau_2Nrg^*bcl(A) \subseteq Y$ whenever $A \subseteq Y$ and Y is Nano open in U . Hence A is $\tau_1\tau_2Nrg^{**}b$ -closed in U .

Theorem 4.2 If A is $\tau_1\tau_2Nrg^{**}b$ -closed and $A \subseteq A' \subseteq \tau_2Nrg^*bcl(A)$ then A' is also $\tau_1\tau_2Nrg^{**}b$ -closed.

Proof. Given that A is $\tau_1\tau_2Nrg^{**}b$ -closed, $\tau_2Nrg^*bcl(A) \subseteq Y$ whenever $A \subseteq Y$ and Y is τ_1 Nano open in U . Also it is given that $A \subseteq A' \subseteq \tau_2Nrg^*bcl(A)$.

To prove. A' is $\tau_1\tau_2Nrg^{**}b$ -closed in U . Let $A' \subseteq Y$ and Y is τ_1 Nano open in U . As $A \subseteq Y$ and A is $\tau_1\tau_2Nrg^{**}b$ -closed in U , $\tau_2Nrg^*bcl(A) \subseteq Y$. But it is given that $A' \subseteq \tau_2Nrg^*bcl(A)$ which implies $\tau_2Nrg^*bcl(A') \subseteq \tau_2Nrg^*bcl(A) \subseteq Y$. Therefore $\tau_2Nrg^*bcl(A') \subseteq Y$ whenever $A' \subseteq Y$ and Y is τ_1 Nano open in U . Hence A' is also $\tau_1\tau_2Nrg^{**}b$ -closed in U .

Remark 4.3 $\tau_2Nrg^*bcl(U - A) \subseteq U - U' \Rightarrow \tau_2Nrg^*bcl(A^c) \subseteq U'^c$ that is $\tau_2Nrg^*bcl(A^c) \subseteq U'^c$ whenever $A^c \subseteq U'^c$ and U'^c is τ_1 Nano open in U . Therefore A^c is $\tau_1\tau_2Nrg^{**}b$ -closed in U .

Theorem 4.4 A set A is $\tau_1\tau_2Nrg^{**}b$ -open in U if and only if $U' \subseteq \tau_1Nrg^*bint(A)$ whenever U' is τ_2 Nano closed set and $U' \subseteq A$.

Proof. Given that A is $\tau_1\tau_2Nrg^{**}b$ -open in U , A^c is $\tau_1\tau_2Nrg^{**}b$ -closed in U .

To prove. $U' \subseteq \tau_1Nrg^*bint(A)$ whenever $U' \subseteq A$ and U' is τ_2 Nano closed in U .

Let U' is τ_2 Nano closed set and $U' \subseteq A$ and $U - A \subseteq U - U'$. Therefore $\tau_2 Nrg^* bcl(U - A) \subseteq U - U'$, as $U - A$ is $\tau_1 \tau_2 Nrg^{**} b$ -closed and $U - U'$ is τ_1 Nano open. By remark(4.3), $U - \tau_1 Nrg^* bint(A) \subseteq U - U'$. Hence $U' \subseteq \tau_1 Nrg^* bint(A)$.

Conversely, we assume that $U' \subseteq \tau_1 Nrg^* bint(A)$ whenever U' is τ_2 Nano closed set and $U' \subseteq A$. Let $A^c \subseteq U'^c$ and U'^c is τ_1 Nano open in U . Now $U' \subseteq \tau_1 Nrg^* bint A$

$$\Rightarrow U - U' \supseteq U - \tau_1 Nrg^* bint(A)$$

$$\Rightarrow U - \tau_1 Nrg^* bint(A) \subseteq U - U'.$$

By Remark(4.3), $\tau_2 Nrg^* bcl(U - A) \subseteq U - U' \Rightarrow \tau_2 Nrg^* bcl(A^c) \subseteq U'^c$ that is $\tau_2 Nrg^* bcl(A^c) \subseteq U'^c$ whenever $A^c \subseteq U'^c$ and U'^c is τ_1 Nano open in U . Therefore A^c is $\tau_1 \tau_2 Nrg^{**} b$ -closed in U . Hence A is $\tau_1 \tau_2 Nrg^{**} b$ -open in U .

5. REFERENCES.

- [1] LellisThivagar M and Carmel Richard, On Nano forms of weakly open sets, International Journal of Mathematics and statistics Invention, 1(1), (2013), 31-37.
- [2] LellisThivagar M and Carmel Richard, On Nano Continuity, Mathematical Theory and Modelling, 3(7), (2013), 32-37.
- [3] Levin N, Generalized closed sets in topology, Rend. Circ. Mat. Palermo, 19(2), (1970), 89-96.
- [4].M.Lellis Thivagar and Carmel Richard, Note on nano topological spaces, Communicated.
- [5] Pawlak Z, Rough sets, International journal of information and computer sciences, 11,(1982), 341-356.
- [6]Reilly and. Vamanamurthy, On α -sets in topological spaces, Tamkang J.Math., 16, 1985, (7-11).
- [7] Sindu K, Indirani K, On regular generalized star b-closed sets in topological spaces., IJMA,4(10),2013,85-92
- [8]Smitha M.G, Indirani K, On regular generalized star b-closed sets in nano topological spaces., International Journal of Physics and Mathematics sciences,5(3),2015,22-26
- [9]Vasanthakannan G, Indirani K, Nano regular generalized star star b-closed sets in nanotopological spaces., International journal of Mathematical archieve.,7(10),2016,115-122