Studies on Novikov identities over the non-associative rings

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Abstract In this paper we studied about the Novikov identities over the non-associative rings with the assumption of 2-divisible and 3-divisible prime alternative rings and also verified whether A 2- divisible ring is prime alternative that satisfy weak identity of Novikov which can be either associative or $N^2 = 0$.

Keywords**—** Novikov identities, non-associative rings, prime alternative rings, divisible ring, weak identity

INTRODUCTION

The study over the Novikov are carried out initially by Osborn, Smith and Kleinfeld. The Novikov identities are satisfied through the rings as follows:

(i) a(bc) = b(ac)

(ii) (a,b,c) = (a,c,b) are known as Novikov rings.

The identities can be varied by substitution process with (ab)c=(ac)b and replacing with the identity of right alternative as (a,b,c) = -(a,c,b) or by the flexible identity (a,b,c) =0. Kleinfeld [8] validated that a 2- divisible simple alternative ring which has no nilpotent elements other than zero has no proper divisors of zero, hence is either associative or an 8- dimensional Cayley - Dickson Algebra. A more general outcome is the usage of ring which is a prime alternative ring that can be associative or a subring of a Cayley - Dickson Algebra, but the proof is complicated.

Irvin Roy Hentzel et al., [3] studied a right alternative ring is a non-associative ring which satisfies the condition (xa) a = x (aa), for all elements x and a. It is assumed that all rings will have a characteristics prime to 2 and 3.

The Artin theorem stated that a pair of elements present in an alternative ring that can constantly yield an associative sub ring. For proving the above statement, it is enough if we prove that a prime alternative ring which has no nilpotent elements other than zero has no proper divisors of zero. In the following section, a simple proof is presented result obtained through Kleinfeld [9].

S.V Pchelincev [6] has given an illustration of a right alternative right nilpotent ring with N is the subset of R^2, which was neither nilpotent nor alternative. M.Slinko [7] has provided a right alternative ring illustration with N is the subset of R^2 that was neither an alternative nor left nilpotent.

MAIN SECTION

The simple right alternative rings are studied by Kleinfeld and Smith [11] with the assumption that either commutators are in the nucleus or all squares x^2 are in the nucleus in order to see that whether these rings are alternative or associative. For proving that for a simple ring of right alternative R with (ab)c = (ac)b, the squared value every element is found in the nucleus of ring R and can be proved as follows:

Consider the ring R to be a 2-divisible non-associative right alternative which satisfies the following expression as:

(ab)c = (ac)b.

R is defined as simple if whenever I is considered as an R ideal then either I = 0 or I = R.

In a right alternative ring, the following identities hold.

1. ((w, x, (y, z) + wx. y. z)) = (w, y, z) x + w (x, y, z)
2. (w, x, yz) + (w,x), y, z) = (w, x, y) z +y(w, x, z)
3. (x, x, x) = 0

Lemma -1:

The set T can be defined as {t∈R/Rt is 0} is a R ideal.

Proof : Apparently T is considered as a left ideal, because the value of RT is 0. Consider that t belongs to T, R contains x, y. Then y(tx) = y(tx + xt) = (yt)x + (yx)t. But (yt)x + (yx)t = 0.

Thus y(tx) = 0 and hence TR ∈ T. Hence T is an R ideal.

Lemma-2:

Whether R is 2- and 3- divisible prime right alternative ring,

R is strongly (-1,1)or C=U

The Proof : As R is primary, under lemma 1

Now, C = U for any 3- divisible other ring. Also, for (R,(R,R),V) - 0, then (R,R,V) c V

If (R,R,V) ⊆ V, it gives ((R,R),R) ⊂ K. Since R is prime, and ((R,R),R)⊂K = 0 or. If $U \subset U_N$ and N = constant. Therefore, repeatedly, R is powerfully (1,-1) or C=U, which satisfies and explains the proposed Lemma.

Theorem-1:

A 2- divisible ring is prime alternative that satisfy weak identity of Novikov which can be either associative or N^2 = 0,

For the alternative ring

$(y^2,x,z) = (x,z,y^2) = (x,z,y)\, y + y(x,z,y)$

$(x,z,y^2) = y(x,z,y)$.

With comparison

(x,z,y) y = 0, so that (y,x,z) y = 0.

From the identity based on alternative ring

0 = (y,x,z)y = (x,y,yz).

y (y,x,z) = 0. we get

$(y^2,x,z) = y(y,x,z) + (y,x,z)x = 0$.

For all x^2belong to N, and x belong to R.

If a, b belong to R, then

$(ab + ba) = (a+b)^2 - a^2 - b^2$ belong to N.

Let n', n belong to N and R contains x,y,z. Subsequently, (n (xn' + nx), z, y) = 0,

so that (n (xn' + nx), z, y) = - (n’ (xn), z, y).

Similarly (n’(nx), z, y) = - (n (xn), z, y) and ((nn)x, z, y) = - (x(nn),z,y).

By combining these three equalities it follows that

((nn')x, y, z) = ((xn) n, y, z) = - ((nn')x, y, z)

This implies, 2 ((nn‘)x, y, z) = 0. Then ((nn) x, y, z) = 0, since R is 2-

divisible. We know that if n e N,

(nx, y, z) = n (x, y, z).

So that (x(nn'), z, y) = nn' (x, z,y) = 0, then N^2 (R,R,R) = 0.

Let I be the N^2generated ideal, and J be the all associators (R,R,R)generated ideal. We note that

x(nn') = n (xn’+n'x) - (xn+nx) n + (nn)x, so that

N^2R is the subset of $RN^2 + N^2$.

Consequently I = $RN^2 + N^2$in an arbitrary ring

J =(R,R,R) R + (R,R,R) .

It follows from N^2 (R,R,R) = 0, that IJ = 0, Since R is prime either

I =0 or J = 0. If I = 0, then N2 = 0. On the other hand if J = 0 then R is associative.

This completes the proof of Theorem.

Lemma- 3: Keep R as a 2- divisible prime correct substitute ring for (xy)z = (xz)y.

Whether R is a correct nilpotent then R is nilpotent.

Proof :

Here we use the property that x^2∈ V to prove the theorem. Clearly (V,R) = 0 and xy + yx∈ V imply it is model like R. Then R/V can be termed as correct variable and anti commutative. Hence R/V is alternative. But then R/V can be alternative and nilpotentimply [10] that R/U is nilpotent. Suppose R to be right nilpotent of index m. Thus, R is right and left nilpotent, hence nilpotent. The same explains the validity of the Lemma.

Lemma -4:

A 3- and 2- divident prime alternative ring filling weak Novikov identity and (x,y,z) + (y,z,x) +(z,x,y) is said to be the center, can be Lie ring or associative.

Proof :

Let R be a 2- divisible prime alternative ring satisfying (x,y,z) + (y,z,x) + (z,x,y) ∈N, to all the terms ∈ R. Suppose N * R, hence $x^2 = 0$, for all x ∈ R. For any n ∈N, (nx)y = n(xy) = -x(ny) = (-xn)y = (xy)n = x(yn) ; - (xy)n = n(xy), so that 2 n(xy) = 0.

This implies n(xy) = 0. Hence $NR^2 = 0$. Set W of all w ∈R such that w R = 0. As NR belong to W weachieve primary NR = 0, for which N = 0. For all anti-commutative ring, (xy)z - x(yz) + (yz)x – y

Theorem-2:.

Keep R as 2-divisible major right alternative ring with (xy)z = (xz)y. If R is considered as true nilpotent then obviously R is nilpotent.

Proof :

W.K.T x^2belong to N and N is an ideal of R. Using this property we prove the theorem.

Now since in the right alternative ring R/N each $x^2 = 0$, R/N is alternative. This means the right nilpotent ring R/N is in fact nilpotent. If o is expresses as the minimum natural number then R is said to be solvable of index m. Since right nilpotent implies solvable, we now prove the theorem by induction on m. If (U) = 0, R = 0. Thus by inductive hypothesis we may assume R^2 is said to be nil-potent, since R^2 is correct nil-potent and has solvable index one less than R. Suppose that $(R^2)n = 0$.

Since R/N is nilpotent , RL* . The N is a model confined in N,

hence for 2n factors of R, it has R (R.....R(RN).......) =R^2 (R.....R(RN)......) = ...= R^2 (R^2 ....

R^2 (R^2N)......) = $(R^2)^2$(R^2...... R^2 (R^2N)......) =.............= (.........($(R^2)^2$ R^2 ,...) R^2)N is the subset of (R^2)°N = 0.

Hence R is left nilpotent. Again a correct substitute ring which is right and left nil-potent must be nil-potent. The above mentioned confirms the initiation and the validity of the theorem.

(zx) + (zx)y -z (xy) + (x,y,z) + (y,z,x) + (z,x,y) = = 2 ((xy)z + (yz)x + (zx)y) which matches double the Jacobian of x,y,z respectively. Hence because of (ii), we conclude the Jacobi identity holds and also R will be Lie ring.

Let R be a 2- dividable principal alternative ring

Filling feeble Novikov identity, y (w,x,z) = (t,x,yz) Then either R is

$r^2 = 0$, for all r ∈ R.

The Proof : From above lemma r^2 ∈N under r∈R. We can take the instance N * R and hence it satisfies the lemma that $N_2 = 0$.

Let K = NR+N.

Since rn = (rn + nr) - nr ∈ K and;

(sn) r = (sn + ns)r - (ns)r ∈ K, for all n ∈ N and r,s, ∈R, K need to be satisfying R be an ideal of R.

Also for n∈N, n(rn') = n (rn1 + n'r) - n(n'r) = 0, because $N^2 = 0$.

Which implies $K^2 = 0$.

Also K = 0 and R is primary. Yet N = 0, therefore, $r^2 = 0$, to all r ∈ R. The above explanation validates the theorem and proves it.

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